Mr. Michael T. Davis WLPCS Calculus	1.6 Trigonometry (3) October 12, 2017

Name: Solutions / Answers

Calculator Not Allowed

1. Convert each angle measure from radians to degrees or vice versa:

a. $\frac{4\pi}{3} \rightarrow$ $\frac{4 \cdot 180^\circ}{3} = 4 \cdot 60^\circ = 240^\circ$

b. $150^\circ \rightarrow$ $150^\circ \cdot \frac{\pi}{180^\circ} = \frac{15\pi}{18} = \frac{5\pi}{6}$

c. $\frac{5\pi}{4} \rightarrow$ $\frac{5 \cdot 180^\circ}{4} = 5 \cdot 45^\circ = 225^\circ$

d. $180^\circ \rightarrow$ π

2. Write each trigonometric expression as an equivalent trigonometric expression in terms of a reference angle. Do not evaluate each expression.

a. $\sin 260^\circ =$ $-\sin 80^\circ$

b. $\cos 100^\circ =$ $-\cos 80^\circ$

c. $\cot \frac{11\pi}{6} =$ $-\cot \frac{\pi}{6}$

3. Evaluate each trigonometric expression.

a. $\sin\frac{\pi}{2} =$ 1

b. $\cot 0 =$ $\frac{1}{0} =$ undefined

c. $\tan\frac{3\pi}{2} =$ undefined

d. $\cos\pi =$ -1

e. $\sec\frac{5\pi}{6} =$ $\dfrac{1}{\cos\frac{5\pi}{6}} = \dfrac{1}{-\frac{\sqrt{3}}{2}} = \dfrac{-2}{\sqrt{3}} = \dfrac{-2\sqrt{3}}{3}$

f. $\csc\frac{4\pi}{3} =$ $\dfrac{1}{\sin\frac{4\pi}{3}} = \dfrac{1}{-\frac{\sqrt{3}}{2}} = \dfrac{-2}{\sqrt{3}} = \dfrac{-2\sqrt{3}}{3}$

4. Evaluate each inverse trigonometric expression (give both degrees and radians).

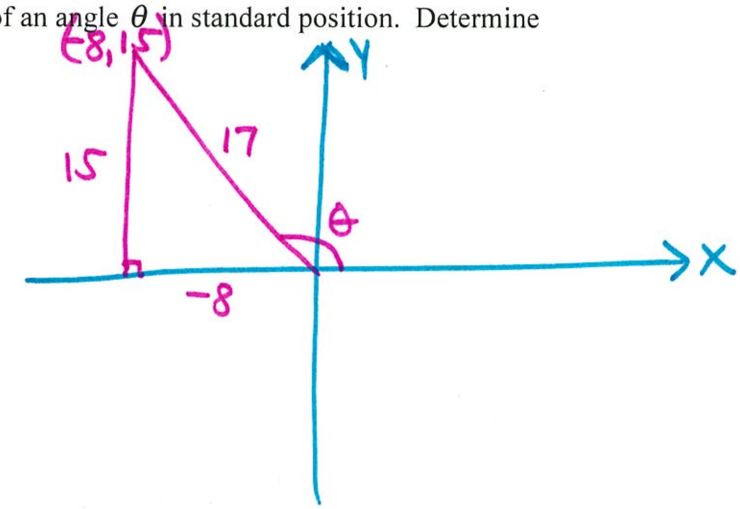
a. $\cos^{-1}\left(\frac{-1}{2}\right)$ $= 120^\circ$ or $\frac{2\pi}{3}$

b. $\sin^{-1}(-1) =$ $-\frac{\pi}{2}$

c. $\tan^{-1}(1) = \frac{\pi}{4}$

d. $\sin^{-1}(0) = 0$

5. The point $P(-8,15)$ lies on the terminal side of an angle θ in standard position. Determine the values of all six trigonometric ratios.



a. $\sin\theta = \frac{15}{17}$

b. $\cos\theta = \frac{-8}{17}$

c. $\tan\theta = \frac{15}{-8}$

d. $\csc\theta = \frac{17}{15}$

e. $\sec\theta = \frac{-17}{8}$

f. $\cot\theta = \frac{-8}{15}$

Calculator Allowed

6. Use your calculator to evaluate and give a decimal approximation for each expression.

a. $\sec 5 =$ 3,525

b. $\csc 50^\circ =$ 1,305

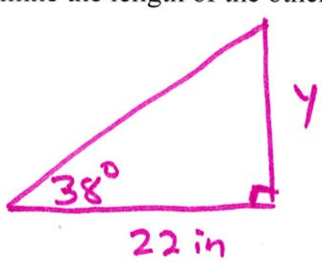
c. $\cot 0.001^\circ =$ 57295.780

d. $\tan 1 =$ 1.557

e. Give in degrees: $\cos^{-1}(0.45) =$ 63.256

f. Give in radians: $\cot^{-1}(11) =$ $\tan^{-1}\left(\frac{1}{11}\right) = 0.0907$

7. A right triangle has an angle measure of 38° and an adjacent leg of length 22 inches. Determine the length of the other leg.

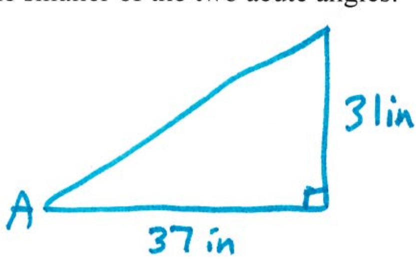


$\tan 38^\circ = \frac{y}{22}$

$22 \tan 38^\circ = y$

$17.188 \approx y$

8. A right triangle has a leg of length 37 cm and a leg of length 31 cm. Determine the measure of the smaller of the two acute angles.

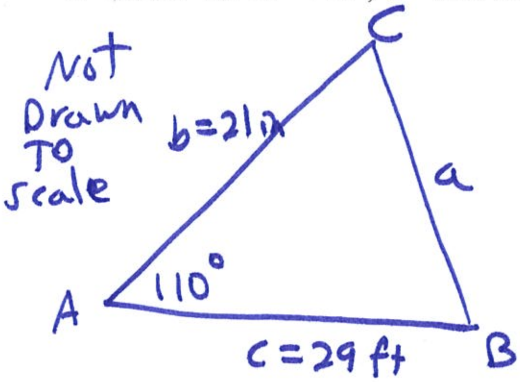


$\tan A = \frac{31}{37}$

$A = \tan^{-1}\left(\frac{31}{37}\right)$

$A \approx 39.958^\circ$

9. ΔABC has $A = 110^{\circ}$, $b = 21$ ft and $c = 29$ ft. Determine the length of side a.

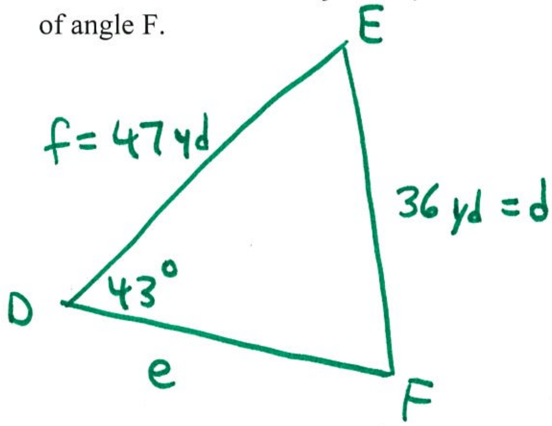


$a^2 = 21^2 + 29^2 - 2(21)(29)\cos 110^{\circ}$

$a^2 \approx 1698.581$

$a \approx 41.214$

10. ΔDEF has $D = 43^{\circ}$, $f = 47$ yards and $d = 36$ yards. Determine the two possible measures of angle F.



$\dfrac{\sin 43^{\circ}}{36} = \dfrac{\sin F}{47}$

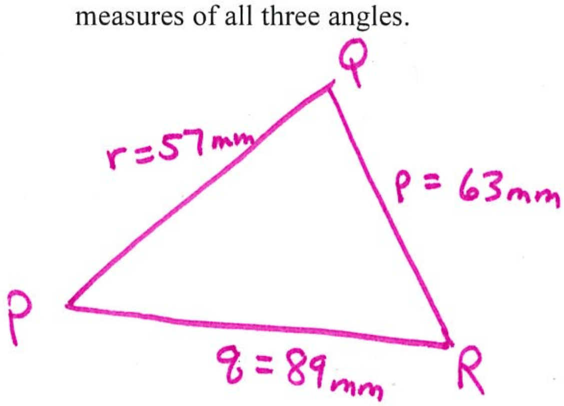
$\dfrac{47 \sin 43^{\circ}}{36} = \sin F$

$0.8904 \approx \sin F$

$62.922^{\circ} \approx F$ or

$180^{\circ} - 62.922^{\circ} = 117.078^{\circ} \approx F$

11. ΔPQR has $r = 57$ mm, $q = 89$ mm and $p = 63$ mm. Solve the triangle by finding the measures of all three angles.



$63^2 = 57^2 + 89^2 - 2(57)(89)\cos P$

$\dfrac{63^2 - 57^2 - 89^2}{-2(57)(89)} = \cos P$

$0.70974 \approx \cos P$

$44.786^{\circ} \approx P$

$57^2 = 63^2 + 89^2 - 2(63)(89)\cos R$

$0.77055 \approx \cos R$

$39.596^{\circ} \approx R$

$Q = 180^{\circ} - P - R$

$Q = 95.617^{\circ}$

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1. Determine the amplitude, period, phase shift, vertical shift, maximum value and minimum value of each sinusoidal function.

a. $f(x) = 7\sin\left(2x - \frac{\pi}{3}\right) - 1$

amplitude : 7
period : π
phase shift : $\frac{\pi}{3}$ to the right
vertical shift : 1 Down
Max : 6
Min : -8

b. $g(x) = \frac{-3}{2}\cos\left(\frac{\pi}{2}x\right)$

amplitude : $\frac{3}{2}$
period : 4
phase shift : None
vertical shift : None
Max : $\frac{3}{2}$
Min : $-\frac{3}{2}$

2. Simplify the each expression.

a. $\cot A \cdot \sec A \cdot \sin A = \frac{\cos A}{\sin A} \cdot \frac{1}{\cos A} \cdot \sin A = 1$

b. $\frac{\sin x \cdot \cos x}{1 - \cos^2 x} = \frac{\sin x \cos x}{\sin^2 x} = \frac{\cos x}{\sin x} = \cot x$

3. Using your graphing calculator, solve $\sin(2x) = \cos\left(x - \frac{\pi}{4}\right)$ given $0 \le x < 2\pi$.

$Y_1 = \sin(2x)$

$Y_2 = \cos(x - \frac{\pi}{4})$ $\quad \{0.7854, 2.8798, 4.9742\}$

4. Verify or prove each identity.

a. $\sin^3\theta + \sin\theta \cdot \cos^2\theta = \sin\theta$

$\sin\theta(\sin^2\theta + \cos^2\theta) = ''$

$\sin\theta(1) = ''$

$\sin\theta = ''$

b. $\dfrac{\cot\alpha\left(1+\tan^2\alpha\right)}{\tan\alpha} = \csc^2\alpha$

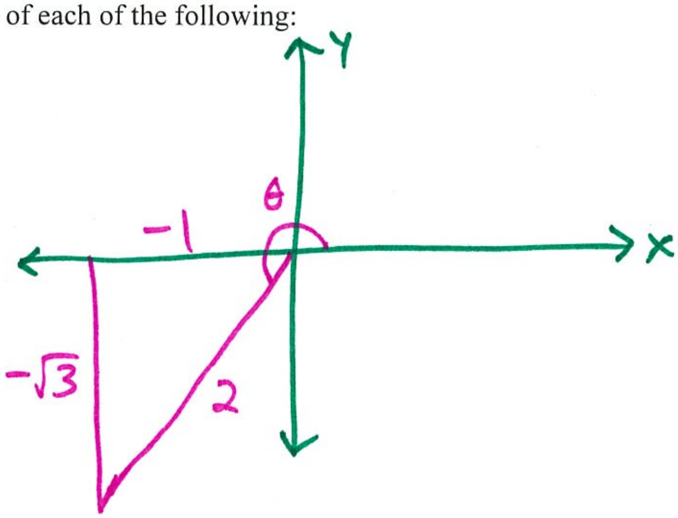
$\dfrac{\frac{\cos\alpha}{\sin\alpha}\cdot\sec^2\alpha}{\frac{\sin\alpha}{\cos\alpha}} = ''$

$\dfrac{\cancel{\cos\alpha}}{\sin\alpha}\cdot\dfrac{1}{\cancel{\cos^2\theta}}\cdot\dfrac{\cancel{\cos\alpha}}{\sin\alpha} = ''$

$\dfrac{1}{\sin^2\alpha} = ''$

$\csc^2\alpha = ''$

5. Given $\csc\theta = \frac{-2}{\sqrt{3}}$ and $\pi < \theta < \frac{3\pi}{2}$, find the value of each of the following:

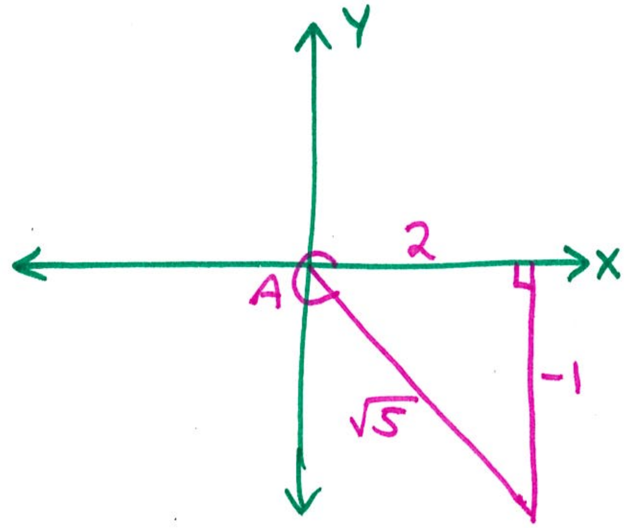


a. $\sec\theta = \frac{2}{-1} = -2$

b. $\sin\theta = \frac{-\sqrt{3}}{2}$

c. $\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

6. Given $\tan A = \frac{-1}{2}$ and $\frac{3\pi}{2} < A < 2\pi$. Find $\sin(2A)$.



$\sin(2A) = 2\sin A\cos A$

$= 2\left(\frac{-1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)$

$= \frac{-4}{5}$

Calculator Not Allowed

7. Give the general solution set for $2\sin^2 x - 1 = 0$.

$2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$\left\{\frac{\pi}{4} + \pi n,\ \frac{3\pi}{4} + \pi n\right\}$

8. Solve $\sin^2 x - \sin x = \cos^2 x$ for $0 \le x < 2\pi$.

$\sin^2 x - \sin x = 1 - \sin^2 x$

$2\sin^2 x - \sin x = 1$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x + 1 = 0 \qquad \sin x - 1 = 0$

$\sin x = \frac{-1}{2} \qquad \sin x = 1$

$\left\{\frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{11\pi}{6}\right\}$

9. Using the formula for the sine of a sum, find an exact value of $\sin 15^\circ$.

$\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ\cos 30^\circ - \sin 30^\circ\cos 45^\circ$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

10. Verify or prove each identity:

a. $\sin\left(\frac{\pi}{2}-x\right)=\cos x$

$\sin\frac{\pi}{2}\cos x - \sin x\cos\frac{\pi}{2} = \cos x$

$(1)\cos x - \sin x\,(0) = \cos x$

$\cos x = \cos x$

b. $\cot B\left(secB - cosB\right)=\sin B$

$\frac{\cos B}{\sin B}\left(\frac{1}{\cos B} - \cos B\right) = "$

$\frac{1}{\sin B} - \frac{\cos^2 B}{\sin B} = "$

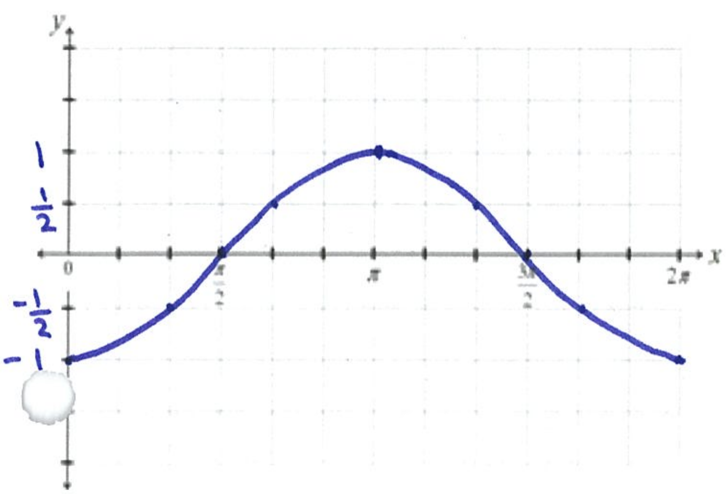
$\frac{1-\cos^2 B}{\sin B} = "$

$\frac{\sin^2 B}{\sin B} = "$

$\sin B = "$

11. Graph each sinusoidal function neatly on the interval $0 \le x \le 2\pi$.

$f(x)=\sin\left(x-\frac{\pi}{2}\right)$



$f(x)=-\cos(x)+1$